

# Gravitational perturbations of the Higgs field

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We study the possible effects of classical gravitational fields on the Higgs vacuum expectation value through the modifications induced in the one-loop effective potential. We concentrate our study on the Higgs self-interactions contribution in a perturbed FRW background. For weak and slowly-varying gravitational fields, a complete set of mode solutions for the Klein-Gordon equation is obtained to leading order in the adiabatic approximation. The mode integrations are calculated using standard dimensional regularization techniques. As expected, the regularized effective potential contains the same divergences as in flat space-time, which can be renormalized without the need of additional counterterms. However, we find new finite non-local contributions which depend on the gravitational potentials, and introduce an explicit space-time dependence on the Higgs potential coefficients. Being finite, the new terms are free of renormalization ambiguities. Inhomogeneities in the effective potential translate into perturbations of the Higgs vacuum expectation value that can have observable effects both on cosmological scales and within the Solar System.

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## I. INTRODUCTION

There are two equally fundamental aspects of the Higgs mechanism for electroweak symmetry breaking which have received remarkably different attention in the last years. On one hand, we have the prediction that a new scalar boson should be present in the spectrum of the theory. Such a new particle has been recently discovered by the ATLAS and CMS experiments at the LHC [1, 2]. The most precise measurement to date of its mass comes from a combination of data from both experiments and is given by  $m_H = 125.09 \pm 0.21(\text{stat}) \pm 0.11(\text{syst})$  GeV [3]. A large deal of experimental effort is being devoted to the study of the properties of the Higgs boson. Apart from improving the precision in the determination of its mass, measurements of its production and decay channels, self-coupling and couplings to other particles are being performed. So far all of them are in excellent agreement with the predictions of the Standard Model (SM) [4–6].

On the other hand, the mechanism also predicts the existence of a Higgs field, i.e. a constant classical field  $\hat{\phi} = v$  with  $v$  the Higgs vacuum expectation value (VEV)<sup>1</sup> given by  $v = 246.221 \pm 0.002$  GeV [7]. It is precisely the interaction with the Higgs field what generates the masses of quarks, leptons and gauge bosons. The presence of this non-vanishing field which permeates all of space is a distinctive feature with respect to the rest of

SM fields which have vanishing VEVs. Moreover, together with the homogeneous gravitational field created by the cosmological energy density, the Higgs field is the only SM field which is present today in the Universe on its largest scales. This fact opens the interesting possibility of probing the Higgs field not only by exciting its quanta in colliders, but by directly perturbing its VEV. Thus, for example, the fact that the Higgs field is a dynamical field sourced by massive particles suggests that the presence of a heavy particle could induce shifts in the masses of neighbouring ones [8]. This effect does not need the production of on-shell Higgs particles, but because of the short range of the corresponding Yukawa interaction, it is negligible at distances beyond the Compton wavelength of the Higgs boson. Existing data does not seem to contain enough kinematic information in order to confirm or exclude it. A similar approach has been proposed in [9] in order to probe the Higgs couplings to electrons and light quarks. The idea of generating peculiar Higgs shifts was also considered in a different context in [10]. In that work a non-minimal coupling of the Higgs field to the space-time curvature was considered. The non-minimal coupling modifies the effective potential inducing shifts of the VEV in high-curvature regions such as those near neutron stars or black holes [11].

In this work, we explore further the effects of classical gravitational fields on the Higgs VEV. We consider the SM Higgs minimally coupled to gravity. The Higgs VEV corresponds to the constant field configuration that minimizes the effective potential. This potential contains not only the classical (tree-level) contribution, but also loop corrections introduced by quantum effects of all the particles that couple to the Higgs, including the Higgs self-interactions [12]. More relevant from the point of view of the present paper is the fact that these quantum

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<sup>1</sup> In the SM, the Higgs VEV is related to the Fermi coupling constant by  $v = (\sqrt{2}G_F)^{-1/2}$ . The value of this constant is known since the original works of Fermi in the early 30’s.

corrections are sensitive to the space-time geometry. The aim of this work is precisely to compute the Higgs one-loop effective potential in weak and slowly-varying gravitational backgrounds. For simplicity and as a first step, we limit ourselves to the contributions of the Higgs self-interactions. The fact that we assume weak gravitational backgrounds, i.e. whose curvature scale is much smaller than the Higgs mass, allows us to use an adiabatic approximation and avoid the problems generated by mode mixing and particle production typical of quantum field theory in curved space-time. For the same reason, we can still define an effective quasi-potential [13, 14] instead of using the full effective action since all the kinetic terms are suppressed with respect to the potential ones.

Our work deals with the calculation of vacuum expectation values of quadratic operators in curved space-time [15, 16]. These are divergent objects whose renormalization requires the introduction of additional counterterms depending on the curvature tensors. Different techniques have been used in the literature to work out these divergences which, because of the fact that they are determined by the short-distance physics, depend locally on the geometry of space-time [17–26]. But, apart from the local divergent contributions, there are also finite non-local terms which are sensitive to the large-scale structure of the manifold and, in general, depend on the quantum state on which the expectation value is evaluated. In some particular simple geometries, such as conformally flat metrics, these finite contributions can be exactly computed in some cases from the knowledge of the trace anomaly, but in general only brute force methods, such as mode summation, are available to evaluate them [27–30]. This is precisely the approach we follow in this work. In particular, we extend the analysis performed in [31] to arbitrary dimension in order to calculate the integrals over the quantum modes using dimensional regularization. The use of dimensional regularization instead of a cutoff as in that paper allows us, in turn, to compute not only the divergences but also the finite parts of the integrals. In addition, several errors in [31] are also corrected in the present paper.

The work is organized as follows: in Section II, the main field equations are given in a  $(D+1)$ -dimensional space-time. The field quantization in the adiabatic approximation is discussed in Section III. Section IV contains the full mode solutions to first order in metric perturbations. The general results for the Higgs effective potential and the method used to obtain them are described in Section V. These results are applied to both static and expanding geometries in Section VI. The paper ends in Section VII with some discussions and conclusions.

## II. BACKGROUND METRIC AND FIELD EQUATIONS

Consider a  $(D+1)$ -dimensional space-time metric which can be written as a scalar perturbation around

a flat Robertson-Walker background

$$ds^2 = a^2(\eta) \{ [1 + 2\Phi(\eta, \mathbf{x})] d\eta^2 - [1 - 2\Psi(\eta, \mathbf{x})] d\mathbf{x}^2 \} \quad (1)$$

where  $\eta$  is the conformal time,  $a(\eta)$  the scale factor, and  $\Phi$  and  $\Psi$  are the scalar perturbations in the longitudinal gauge. This metric describes the space-time geometry in cosmological contexts with density perturbations, but also, in the  $a(\eta) = 1$  case, it provides a good description of weak gravitational fields generated by slowly-rotating astrophysical objects like the Sun.

The action for a minimally coupled real scalar field with potential  $V(\phi)$  in arbitrary  $(D+1)$ -dimensional curved space-time reads

$$S[\phi] = \int d^{D+1}x \sqrt{g} \left( \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right). \quad (2)$$

In the case of the real Higgs field, the classical potential is given by

$$V(\phi) = V_0 + \frac{1}{2} M^2 \phi^2 + \frac{\lambda}{4} \phi^4 \quad (3)$$

with  $M^2 < 0$ . The minimum corresponds to  $\phi = v$  with  $v^2 = -M^2/\lambda$ . The mass of the Higgs boson at tree-level is given by  $m_H^2 = V''(v) = -2M^2$  and from the recently measured value of  $m_H$  at the LHC, the Higgs self-coupling is  $\lambda \simeq 1/8$ .

The equation of motion derived from the classical action reads

$$\square \phi + V'(\phi) = 0. \quad (4)$$

Up to first order in metric perturbations, it can be written as

$$\phi'' + [(D-1)\mathcal{H} - \Phi' - D\Psi'] \phi' - [1 + 2(\Phi + \Psi)] \nabla^2 \phi - \nabla \phi \cdot \nabla [\Phi - (D-2)\Psi] + a^2(1 + 2\Phi) V'(\phi) = 0 \quad (5)$$

where  $\mathcal{H} = a'/a$  is the comoving Hubble parameter.

The field  $\phi$  can be decomposed in a classical part  $\hat{\phi}(\eta, \mathbf{x})$  and a field  $\delta\phi(\eta, \mathbf{x})$  describing the quantum fluctuations around the classical solution

$$\phi(\eta, \mathbf{x}) = \hat{\phi}(\eta, \mathbf{x}) + \delta\phi(\eta, \mathbf{x}), \quad (6)$$

with  $\hat{\phi}(\eta, \mathbf{x}) = \langle 0 | \phi(\eta, \mathbf{x}) | 0 \rangle$  and  $\langle 0 | \delta\phi(\eta, \mathbf{x}) | 0 \rangle = 0$ .

In general because of the inhomogeneities of the space-time metric, the classical field  $\hat{\phi}$  could acquire a space-time dependence with  $\nabla \hat{\phi} \sim \mathcal{H} \hat{\phi}$  or  $\nabla \hat{\phi} \sim \hat{\phi} \nabla \Phi$ . In the flat space-time limit, the constant VEV is recovered.

The effects of quantum fluctuations on the classical solution can be taken into account by expanding the potential in (5) around  $\hat{\phi}$  and taking the corresponding vacuum expectation value [32]. Thus, the effective equation of motion for the classical field is

$$\hat{\phi}'' + [(D-1)\mathcal{H} - \Phi' - D\Psi'] \hat{\phi}' - [1 + 2(\Phi + \Psi)] \nabla^2 \hat{\phi} - \nabla \hat{\phi} \cdot \nabla [\Phi - (D-2)\Psi] + a^2(1 + 2\Phi) V'_{\text{eff}}(\hat{\phi}) = 0 \quad (7)$$

where cubic and higher-order terms in  $\delta\phi$  have been neglected and we have introduced an effective potential  $V_{\text{eff}}$  given by

$$V'_{\text{eff}}(\hat{\phi}) = V'(\hat{\phi}) + V'_1(\hat{\phi}) \quad (8)$$

with

$$V'_1(\hat{\phi}) = \frac{1}{2} V'''(\hat{\phi}) \langle 0 | \delta\phi^2 | 0 \rangle. \quad (9)$$

On general grounds we expect a large hierarchy between the potential term and the terms involving space-time derivatives in (7). Thus, whereas the size of the potential term is expected to be  $V'_{\text{eff}}(\hat{\phi}) \sim m_H^3$ , the estimates for the field derivatives mentioned before suggest that the other terms will be  $\mathcal{O}(m_H H^2)$  or  $\mathcal{O}(m_H \nabla^2 \Phi)$ . Thus, the derivative terms are suppressed with respect to the potential one by  $\mathcal{O}(H^2/m_H^2)$  or  $\mathcal{O}(\nabla^2 \Phi/m_H^2)$ . Since at late times  $\{H^2, \nabla^2 \Phi\} \ll m_H^2$ , we can safely neglect them and the equation of motion for the classical field reduces to

$$V'_{\text{eff}}(\hat{\phi}) \simeq 0, \quad (10)$$

i.e. the effective (quasi-) potential correctly determines the VEV for a slowly varying background metric.

On the other hand, by linearizing (5), the equation for the fluctuation field  $\delta\phi$  can be obtained

$$\delta\phi'' + [(D-1)\mathcal{H} - \Phi' - D\Psi']\delta\phi' - [1 + 2(\Phi + \Psi)]\nabla^2\delta\phi - \nabla\delta\phi \cdot \nabla[\Phi - (D-2)\Psi] + a^2(1+2\Phi)m^2(\hat{\phi})\delta\phi = 0, \quad (11)$$

where the mass of  $\delta\phi$  is given by

$$m^2(\hat{\phi}) = V''(\hat{\phi}) = M^2 + 3\lambda\hat{\phi}^2. \quad (12)$$

This mass can be considered as constant as far as the classical field solution at tree level is just a constant field satisfying  $V'(\hat{\phi}) = 0$ .

Finally, integrating (9) in  $\hat{\phi}$ , the one-loop contribution to the effective potential can be written as

$$V_1(\hat{\phi}) = \frac{1}{2} \int_0^{m^2(\hat{\phi})} dm^2 \langle 0 | \delta\phi^2 | 0 \rangle. \quad (13)$$

Then, the central object of this calculation is the vacuum expectation value of a quadratic operator.

### III. QUANTIZATION AND ADIABATIC APPROXIMATION

In order to evaluate  $V_1(\hat{\phi})$ , we need to quantize the fluctuation field. Because of the inhomogeneities of the metric tensor, exact solutions for the perturbed equation (11) are not expected to be found. Nevertheless, a perturbative expansion of the solution in powers of metric perturbations can be obtained. Moreover, when the mode frequency  $\omega$  is larger than the typical temporal or

spatial frequency of the background metric, i.e.  $\omega^2 \gg \mathcal{H}^2$  and  $\omega^2 \gg \{\nabla^2 \Phi, \nabla^2 \Psi\}$ , one can consider an adiabatic approximation in order to quantize the field fluctuations  $\delta\phi$ . Since  $\omega \geq m_H$ , the adiabatic approximation is extremely good during the whole matter and acceleration eras until present, and also during most of the radiation era, for all cosmological and astrophysical scales of interest.

Let us start with the canonical quantization procedure for the field perturbations  $\delta\phi$ . Thus, following [33, 34], we build a complete set of mode solutions for (11), which are orthonormal with respect to the standard scalar product in curved space-time [15]

$$(\delta\phi_p, \delta\phi_q) = i \int_{\Sigma} [\delta\phi_q^* (\partial_{\mu} \delta\phi_p) - (\partial_{\mu} \delta\phi_q^*) \delta\phi_p] \sqrt{g_{\Sigma}} d\Sigma^{\mu}, \quad (14)$$

with  $d\Sigma^{\mu} = n^{\mu} d\Sigma$ . Here  $n^{\mu}$  is a unit timelike vector directed to the future and orthogonal to the  $\eta = \text{const.}$  hypersurface  $\Sigma$ , i.e.

$$d\Sigma^{\mu} = d^D \mathbf{x} \left( \frac{1 - \Phi}{a}, \mathbf{0} \right), \quad (15)$$

whereas the determinant of the metric on the spatial hypersurface reads to first order in metric perturbations

$$\sqrt{g_{\Sigma}} = a^D (1 - D\Psi). \quad (16)$$

With this definition, the scalar product is independent on the choice of spatial hypersurface  $\Sigma$ .

In terms of orthonormal modes,

$$(\delta\phi_p, \delta\phi_q) = \delta^D(\mathbf{p} - \mathbf{q}), \quad (17)$$

the fluctuation field  $\delta\phi$  can be expanded as

$$\delta\phi(\eta, \mathbf{x}) = \int d^D \mathbf{k} \left[ a_{\mathbf{k}} \delta\phi_{\mathbf{k}}(\eta, \mathbf{x}) + a_{\mathbf{k}}^{\dagger} \delta\phi_{\mathbf{k}}^*(\eta, \mathbf{x}) \right]. \quad (18)$$

The corresponding creation and annihilation operators satisfy the standard commutation relations

$$[a_{\mathbf{p}}, a_{\mathbf{q}}^{\dagger}] = \delta^D(\mathbf{p} - \mathbf{q}) \quad (19)$$

and the vacuum state associated to the quantum modes  $\{\delta\phi_{\mathbf{k}}\}$  is defined as usual by  $a_{\mathbf{k}} |0\rangle = 0 \forall \mathbf{k}$ .

In order to construct the orthonormal set, we use an WKB ansatz

$$\delta\phi_{\mathbf{k}}(\eta, \mathbf{x}) = f_{\mathbf{k}}(\eta, \mathbf{x}) e^{i\theta_{\mathbf{k}}(\eta, \mathbf{x})}, \quad (20)$$

and assume that  $f_{\mathbf{k}}(\eta, \mathbf{x})$  evolves slowly in space and time, whereas the evolution of  $\theta_{\mathbf{k}}(\eta, \mathbf{x})$  is rapid. In general, as mentioned above, such an adiabatic ansatz works whenever the Compton wavelength of the field perturbation is much smaller than the typical astrophysical or cosmological scales involved. In particular, in the adiabatic expansion we assume  $\partial\theta \sim ma$  and  $\partial f \sim \mathcal{H}f$ .

Substituting (20) in (11), we obtain to the leading adiabatic order  $\mathcal{O}((\partial\theta)^2)$

$$-\theta_{\mathbf{k}}'^2 + [1 + 2(\Phi + \Psi)](\nabla\theta_{\mathbf{k}})^2 + m^2 a^2 (1 + 2\Phi) = 0 \quad (21)$$

and to the next to leading order  $\mathcal{O}(\partial\theta)$

$$\begin{aligned} f_k \theta_k'' + 2f_k' \theta_k' + [(D-1)\mathcal{H} - \Phi' - D\Psi'] f_k \theta_k' \\ - f_k \nabla^2 \theta_k - 2\nabla f_k \cdot \nabla \theta_k \\ - f_k \nabla \theta_k \cdot \nabla [\Phi - (D-2)\Psi] = 0. \end{aligned} \quad (22)$$

Notice that  $\partial^2 \theta \sim \mathcal{H} \partial \theta$  and that, in the adiabatic expansion,  $\mathcal{H} \sim \partial \Phi$ .

#### IV. PERTURBATIVE EXPANSION AND MODE SOLUTIONS

To solve these two equations, (21) and (22), we look for a perturbative expansion in the metric potentials. To obtain the lowest order solution, i.e. in the absence of metric perturbations, we write (11) in the limit  $\Phi = \Psi = 0$  and get

$$\delta\phi^{(0)''} + (D-1)\mathcal{H}\delta\phi^{(0)'} - \nabla^2 \delta\phi^{(0)} + a^2 m^2(\hat{\phi}) \delta\phi^{(0)} = 0, \quad (23)$$

where  $a^2 m^2(\hat{\phi})$  only depends on time. Fourier transforming the spatial coordinates, the following positive frequency solution with momentum  $\mathbf{k}$  is obtained

$$\delta\phi_k^{(0)}(\eta, \mathbf{x}) = F_k(\eta) e^{i\mathbf{k}\cdot\mathbf{x} - i\int^\eta \omega(\eta') d\eta'} \quad (24)$$

with

$$\omega^2 = k^2 + m^2 a^2 \quad (25)$$

and

$$F_k(\eta) = \frac{1}{(2\pi)^{D/2}} \frac{1}{\sqrt{2\omega} a^{(D-1)/2}} \quad (26)$$

which is fixed by the normalization condition (17).

Once the unperturbed solution is known, we can look for the first order corrections. Thus, the amplitude and phase of (20) are expanded in metric perturbations as follows

$$\begin{aligned} f_k(\eta, \mathbf{x}) &= F_k(\eta) + \delta f_k(\eta, \mathbf{x}) \\ \theta_k(\eta, \mathbf{x}) &= \mathbf{k} \cdot \mathbf{x} - \int^\eta \omega(\eta') d\eta' + \delta\theta_k(\eta, \mathbf{x}) \end{aligned} \quad (27)$$

where  $\delta f_k$  and  $\delta\theta_k$  are first order in perturbations. Substituting (27) in the leading equation (21), we obtain (25) to the lowest order as expected, and to first order we get

$$\omega \delta\theta_k' + \mathbf{k} \cdot \nabla \delta\theta_k + k^2(\Phi + \Psi) + m^2 a^2 \Phi = 0. \quad (28)$$

On the other hand, by substituting in the next to leading equation (22), we recover (26) to the lowest perturbative order, whereas to first order we get

$$\begin{aligned} F_k \delta\theta_k'' + 2F_k' \delta\theta_k' + (D-1)\mathcal{H} F_k \delta\theta_k' - F_k \nabla^2 \delta\theta_k \\ - 2\omega \delta f_k' - 2\mathbf{k} \cdot \nabla \delta f_k - (D-1)\omega \mathcal{H} \delta f_k - \omega' \delta f_k \\ + \omega F_k \Phi' + D\omega F_k \Psi' - F_k \mathbf{k} \cdot \nabla [\Phi - (D-2)\Psi] = 0. \end{aligned} \quad (29)$$

The two new equations (28) and (29) can also be solved by performing an additional Fourier transformation in the spatial coordinates since the equations coefficients only depend on time.

#### Phase solution $\delta\theta_k$

Equation (28) in Fourier space reads

$$\delta\theta_k'(\eta, \mathbf{p}) + i \frac{\mathbf{k} \cdot \mathbf{p}}{\omega} \delta\theta_k(\eta, \mathbf{p}) = -\omega \left[ \Phi(\eta, \mathbf{p}) + \frac{k^2}{\omega^2} \Psi(\eta, \mathbf{p}) \right], \quad (30)$$

where

$$\delta\theta_k(\eta, \mathbf{p}) = \frac{1}{(2\pi)^{3/2}} \int d^3 \mathbf{x} \delta\theta_k(\eta, \mathbf{x}) e^{-i\mathbf{p} \cdot \mathbf{x}} \quad (31)$$

and analogous definitions apply for  $\Phi(\eta, \mathbf{p})$ ,  $\Psi(\eta, \mathbf{p})$  and  $\delta f_k(\eta, \mathbf{p})$ .<sup>2</sup> Defining

$$\begin{aligned} \beta_k(\eta_f, \eta_i) &= \int_{\eta_i}^{\eta_f} \frac{d\eta'}{\omega(\eta')} \\ G_k(\eta, \mathbf{p}) &= -\omega \left[ \Phi(\eta, \mathbf{p}) + \frac{k^2}{\omega^2} \Psi(\eta, \mathbf{p}) \right], \end{aligned} \quad (32)$$

the solution of (30) is

$$\delta\theta_k(\eta, \mathbf{p}) = \int_0^\eta e^{-i\mathbf{k} \cdot \mathbf{p} \beta_k(\eta, \eta')} G_k(\eta', \mathbf{p}) d\eta'. \quad (33)$$

Here the integration limits have been chosen in such a way that the perturbed phases match the unperturbed ones at  $\eta = 0$ . This boundary condition defines our vacuum state. See Section VII for further details.

#### Amplitude solution $\delta f_k$

Following a similar procedure with the next to leading order equation (29), it can be rewritten in Fourier space as

$$\begin{aligned} \frac{2\sqrt{\omega}}{a^{(D-1)/2}} e^{-i\mathbf{k} \cdot \mathbf{p} \beta_k(\eta, 0)} (e^{i\mathbf{k} \cdot \mathbf{p} \beta_k(\eta, 0)} a^{(D-2)/2} \sqrt{\omega} \delta f_k(\eta, \mathbf{p}))' \\ = F_k(\eta) H_k(\eta, \mathbf{p}) \end{aligned} \quad (34)$$

where

$$H_k(\eta, \mathbf{p}) = \omega Q_k'(\eta, \mathbf{p}) + T_k(\eta, \mathbf{p}) \quad (35)$$

with

$$Q_k(\eta, \mathbf{p}) = -i \frac{\mathbf{k} \cdot \mathbf{p}}{\omega^2} \delta\theta_k(\eta, \mathbf{p}) + \left[ D - \frac{k^2}{\omega^2} \right] \Psi(\eta, \mathbf{p}) \quad (36)$$

and

$$\begin{aligned} T_k(\eta, \mathbf{p}) &= p^2 \delta\theta_k(\eta, \mathbf{p}) \\ &\quad - i \mathbf{k} \cdot \mathbf{p} [\Phi(\eta, \mathbf{p}) - (D-2)\Psi(\eta, \mathbf{p})]. \end{aligned} \quad (37)$$

The corresponding solution is given by

$$\delta f_k(\eta, \mathbf{p}) = F_k(\eta) P_k(\eta, \mathbf{p}) \quad (38)$$

<sup>2</sup> In the following, the wavevector of the quantum modes is denoted by  $\mathbf{k}$ , and  $\mathbf{p}$  is used for that of metric perturbations.

with

$$P_k(\eta, \mathbf{p}) = \int_0^\eta e^{-i\mathbf{k}\cdot\mathbf{p}\beta_k(\eta, \eta')} \frac{H_k(\eta', \mathbf{p})}{2\omega(\eta')} d\eta' + e^{-i\mathbf{k}\cdot\mathbf{p}\beta_k(\eta, 0)} D_k(\mathbf{p}). \quad (39)$$

The integration constants  $D_k(\mathbf{p})$  are fixed by the normalization condition (17).

#### Time-independent gravitational potentials

For simplicity, in the rest of the work we focus in time-independent gravitational potentials. This case encompasses super-Hubble modes in both matter and radiation era, and also sub-Hubble modes in the matter era. This is also a good approximation to describe the gravitational potentials in the Solar System. In such a case the constants  $D_k(\mathbf{p})$  are given by

$$D_k(\mathbf{p}) = \frac{1}{2} \left( D - \frac{k^2}{\omega_0^2} \right) \Psi(\mathbf{p}) \quad (40)$$

with  $\omega_0 = \omega(\eta)|_{\eta=0}$ .

Integrating by parts in (39), the integration constant can be eliminated and the following expression is obtained

$$P_k(\eta, \mathbf{p}) = \frac{1}{2} Q_k(\eta, \mathbf{p}) - i \int_0^\eta \left\{ \frac{\mathbf{k} \cdot \mathbf{p}}{2\omega(\eta')} e^{-i\mathbf{k}\cdot\mathbf{p}\beta_k(\eta, \eta')} \times \left[ Q_k(\eta', \mathbf{p}) + \frac{T_k(\eta', \mathbf{p})}{\mathbf{k} \cdot \mathbf{p}} \right] \right\} d\eta'. \quad (41)$$

There are three types of contributions to  $P_k$ , depending on the number of time integrals involved. Thus, we can write

$$P_k(\eta, \mathbf{p}) = P_k^{(0)}(\eta, \mathbf{p}) + P_k^{(1)}(\eta, \mathbf{p}) + P_k^{(2)}(\eta, \mathbf{p}) \quad (42)$$

where

$$P_k^{(0)}(\eta, \mathbf{p}) = \frac{1}{2} \left( D - \frac{k^2}{\omega(\eta)^2} \right) \Psi(\mathbf{p}) \quad (43)$$

$$P_k^{(1)}(\eta, \mathbf{p}) = \int_0^\eta e^{-i\mathbf{k}\cdot\mathbf{p}\beta_k(\eta, \eta')} N_k^{(1)}(\eta, \eta', \mathbf{p}) d\eta' \quad (44)$$

$$P_k^{(2)}(\eta, \mathbf{p}) = \int_0^\eta \int_0^{\eta'} e^{-i\mathbf{k}\cdot\mathbf{p}\beta_k(\eta, \eta'')} N_k^{(2)}(\eta', \eta'', \mathbf{p}) d\eta'' d\eta' \quad (45)$$

with

$$N_k^{(1)}(\eta, \eta', \mathbf{p}) = \frac{i\mathbf{k} \cdot \mathbf{p}}{2\omega^2(\eta)\omega(\eta')} \left\{ [\omega^2(\eta') - \omega^2(\eta)] \Phi(\mathbf{p}) + \left[ k^2 + \omega^2(\eta) \left( \frac{k^2}{\omega^2(\eta')} - 2 \right) \right] \Psi(\mathbf{p}) \right\} \quad (46)$$

$$N_k^{(2)}(\eta', \eta'', \mathbf{p}) = \frac{(\mathbf{k} \cdot \mathbf{p})^2 - p^2 \omega^2(\eta')}{2\omega^3(\eta')\omega(\eta'')} \times [\omega^2(\eta'') \Phi(\mathbf{p}) + k^2 \Psi(\mathbf{p})] \quad (47)$$

where  $p = |\mathbf{p}|$ .

## V. HIGGS EFFECTIVE POTENTIAL

Once we have the expressions for the mode solutions of the perturbative equations, namely (33) and (38); we can proceed to calculate the one-loop contribution to the effective potential (13).

Let us first calculate  $\langle 0|\delta\phi^2(\eta, \mathbf{x})|0\rangle$  to first order in metric perturbations. Because of the inhomogeneity of the background, this quantity depends on  $(\eta, \mathbf{x})$  as follows

$$\langle 0|\delta\phi^2(\eta, \mathbf{x})|0\rangle = \Delta_h(\eta) + \Delta_i(\eta, \mathbf{x}) \quad (48)$$

where

$$\Delta_h(\eta) = \int d^D \mathbf{k} F_k^2(\eta) \quad (49)$$

and

$$\Delta_i(\eta, \mathbf{x}) = 2 \int d^D \mathbf{k} F_k^2(\eta) [\text{Re} P_k(\eta, \mathbf{x}) - \text{Im} \delta\theta_k(\eta, \mathbf{x})] \quad (50)$$

#### Homogeneous contribution $\Delta_h(\eta)$

The homogeneous contribution  $\Delta_h$  reads

$$\Delta_h(\eta) = \frac{1}{2(2\pi)^D a^{D-1}(\eta)} \int \frac{d^D \mathbf{k}}{\sqrt{k^2 + m^2 a^2(\eta)}} \quad (51)$$

or

$$\Delta_h(\eta) = \frac{1}{2(2\pi)^D a^{D-1}(\eta)} \frac{2\pi^{D/2}}{\Gamma(D/2)} \int_0^\infty \frac{dk k^{D-1}}{\sqrt{k^2 + m^2 a^2(\eta)}} \quad (52)$$

which is analogous to the Minkowskian result, except for the scale-factor dependence.

#### Non-homogeneous contribution $\Delta_i(\eta, \mathbf{x})$

The inhomogeneous component  $\Delta_i$  can be dealt with more easily in momentum space. The only angular dependence of the quantum fluctuation wavevector  $\mathbf{k}$  enters as  $\mathbf{k} \cdot \mathbf{p} = k p \hat{x}$  with  $\hat{x} = \cos \theta$ , where we have taken the  $k_z$  direction along  $\mathbf{p}$ . On the other hand, the contribution from  $\delta\theta$  in (50) vanishes after integrating in  $\hat{x}$ . Then, we have

$$\Delta_i(\eta, \mathbf{p}) = \frac{1}{(2\pi)^D a^{D-1}(\eta)} \int d^D \mathbf{k} \frac{P_k(\eta, \mathbf{p})}{\sqrt{k^2 + m^2 a^2(\eta)}}. \quad (53)$$

Since the integration on  $\hat{x}$  can be performed in a straightforward way, let us define

$$\hat{P}_k(\eta, \mathbf{p}) = \int_{-1}^1 d\hat{x} P_k(\eta, \mathbf{p}). \quad (54)$$



Hence, we can write (see Appendix A)

$$\Delta_i(\eta, \mathbf{p}) = \frac{1}{(2\pi)^D a^{D-1}(\eta)} \frac{\pi^{D/2}}{\Gamma(D/2)} \times \int_0^\infty dk \frac{k^{D-1} \hat{P}_k(\eta, \mathbf{p})}{\sqrt{k^2 + m^2} a^2(\eta)}. \quad (55)$$

Both integrals (52) and (55) are divergent in  $D = 3$  dimensions and they should be regularized as discussed in next section.

### Regularization

Let us now discuss the regularization procedure based on standard dimensional regularization techniques.

*Regularized homogeneous contribution  $\Delta_h(\eta)$*

The momentum integral in  $\Delta_h$  (52) can be done using (A3) of Appendix A. After expanding for small  $\epsilon$  with  $D = 3 - \epsilon$  dimensions, the final result is

$$\Delta_h(\eta) = \frac{m^2(\hat{\phi})}{16\pi^2} \left[ \ln \left( \frac{m^2(\hat{\phi})}{\mu^2} \right) - N_\epsilon - \frac{3}{2} \right] \quad (56)$$

where

$$N_\epsilon = \frac{2}{\epsilon} + \log 4\pi - \gamma \quad (57)$$

with  $\gamma$  de Euler-Mascheroni constant and  $\mu$  the renormalization scale.

*Regularized non-homogeneous contribution  $\Delta_i(\eta, \mathbf{x})$*

Let us now consider the inhomogeneous contribution (55). We cannot apply directly standard dimensional regularization formulae because of the non-trivial  $k$ -dependence of  $\hat{P}_k(\eta, \mathbf{p})$ . Thus, additional work is necessary.

First, it should be noticed that the dependence of  $\hat{P}_k(\eta, \mathbf{p})$  on the direction of  $\mathbf{p}$  only enters through the potentials,  $\Phi(\mathbf{p})$  and  $\Psi(\mathbf{p})$ . Therefore, it can be expanded in the following way

$$\hat{P}_k(\eta, \mathbf{p}) = \left[ \sum_{l=0}^\infty P_{k,l}^\Phi(\eta) p^{2l} \right] \Phi(\mathbf{p}) + \left[ \sum_{l=0}^\infty P_{k,l}^\Psi(\eta) p^{2l} \right] \Psi(\mathbf{p}). \quad (58)$$

The coefficients  $P_{k,l}^{\{\Phi, \Psi\}}(\eta)$  are given in Appendix B. The  $l = 0$  terms only get contributions from the  $P_k^{(0)}(\eta, \mathbf{p})$  term given in (43) and its integral vanishes in dimensional regularization. The  $l > 0$  terms involve time integrals of the form

$$\int_0^\eta d\eta' \left( \prod_{i=1}^{2l-1} \int_{\eta'}^\eta \frac{d\eta_i}{\omega(\eta_i)} \right) \frac{k^{2\alpha}}{\omega(\eta)^a \omega(\eta')^b} \quad (59)$$

for the contributions coming from  $P_k^{(1)}(\eta, \mathbf{p})$  in (44), and

$$\int_0^\eta d\eta' \int_0^{\eta'} d\eta'' \left( \prod_{i=1}^{2l-2} \int_{\eta''}^\eta \frac{d\eta_i}{\omega(\eta_i)} \right) \frac{k^{2\alpha}}{\omega(\eta)^a \omega(\eta')^b \omega(\eta'')^c} \quad (60)$$

for those coming from  $P_k^{(2)}(\eta, \mathbf{p})$  in (45), with  $\alpha, a, b, c \in \mathbb{Z}$ . In order to simplify the functional dependence on  $k$ , we apply the generalized Feynman trick,

$$\frac{1}{A_1^{d_1} \cdots A_n^{d_n}} = \frac{\Gamma(d_1 + \cdots + d_n)}{\Gamma(d_1) \cdots \Gamma(d_n)} \int_0^1 dx_1 \cdots \int_0^1 dx_n \quad (61)$$

$$\times \delta(x_1 + \cdots + x_n - 1) \frac{x_1^{d_1-1} \cdots x_n^{d_n-1}}{(x_1 A_1 + \cdots + x_n A_n)^{d_1 + \cdots + d_n}}.$$

Then, let the parameters of the Feynman formula be defined by

$$n = 2l + 1 \quad (62)$$

$$A_j = \begin{cases} \omega^2(\eta) & \text{if } j = 1 \\ \omega^2(\eta') & \text{if } j = 2 \\ \omega^2(\eta_{j-2}) & \text{if } 3 \leq j \leq 2l + 1 \end{cases} \quad (63)$$

$$d_j = \begin{cases} a/2 & \text{if } j = 1 \\ b/2 & \text{if } j = 2 \\ 1/2 & \text{if } 3 \leq j \leq 2l + 1 \end{cases} \quad (64)$$

for the case (59) (with a trivial modification for the expression (60)). In this way, the  $k$ -dependence only appears in  $\sum_{i=1}^{2l+1} x_i \omega_i^2$  which can be simplified in the following way

$$\sum_{i=1}^{2l+1} x_i \omega_i^2 = \sum_{i=1}^{2l+1} x_i (k^2 + m^2 a_i^2) = k^2 + m^2 \sum_{i=1}^{2l+1} x_i a_i^2 \quad (65)$$

where we have used  $\sum_{i=1}^{2l+1} x_i = 1$ . Now, the  $k$ -dependence is simple enough to use standard dimensional regularization formulae (Appendix A). The integration over the  $\{x_i\}$  and the time integrals can be performed analitically (Appendix C).

As we did with  $\hat{P}_k(\eta, \mathbf{p})$ , we now decompose  $\Delta_i(\eta, \mathbf{p})$  into two terms proportional to  $\Phi(\mathbf{p})$  and  $\Psi(\mathbf{p})$  respectively

$$\Delta_i(\eta, \mathbf{p}) = \Delta_i^\Phi(\eta, \mathbf{p}) \Phi(\mathbf{p}) + \Delta_i^\Psi(\eta, \mathbf{p}) \Psi(\mathbf{p}). \quad (66)$$

Then, integrating in dimensional regularization, we see that the  $\mathcal{O}(1/\epsilon)$  terms cancel out and the results is finite

$$\Delta_i^{\{\Phi, \Psi\}}(\eta, \mathbf{p}) = \frac{m^2}{4\pi^2 a^2(\eta)} \left[ \sum_{l=1}^\infty R_l^{\{\Phi, \Psi\}}(\eta) p^{2l} \right] \quad (67)$$

where  $R_l^{\{\Phi, \Psi\}}$  are the already-regularized integrals in  $k$  of  $P_{k,l}^{\{\Phi, \Psi\}}$  divided by  $m^2$  for convenience. The coefficients  $R_l^{\{\Phi, \Psi\}}$  can be written as

$$R_l^{\{\Phi, \Psi\}}(\eta) = R_{l, \text{pol}}^{\{\Phi, \Psi\}}(\eta) + R_{l, \text{log}}^{\{\Phi, \Psi\}}(\eta) \quad (68)$$

where, as shown in Appendix C,  $R_{l,\text{pol}}^{\{\Phi,\Psi\}}$  are polynomials in  $\eta$  and  $R_{l,\text{log}}^{\{\Phi,\Psi\}}$  involve a logarithmic dependence on  $\eta$ .

The most important aspect of (67) is that all the divergent parts have cancelled out. In particular, the divergent terms coming from  $P_k^{(1)}(\eta, \mathbf{p})$  cancel exactly the ones from  $P_k^{(2)}(\eta, \mathbf{p})$  order by order in  $p$ . This means that the UV behaviour is the same as in an unperturbed FRW background and the inhomogeneous contributions are finite to the leading adiabatic order.

### Higgs effective potential

Taking into account (13), the one-loop contribution to the effective potential can be expressed as

$$V_1(\eta, \mathbf{x}) = V_1^h(\eta) + V_1^i(\eta, \mathbf{x}). \quad (69)$$

The homogeneous contribution reads

$$V_1^h(\eta) = \frac{1}{2} \int_0^{m^2(\hat{\phi})} dm^2 \Delta_h(\eta) \quad (70)$$

substituting (56), we get

$$V_1^h(\hat{\phi}) = \frac{m^4(\hat{\phi})}{64\pi^2} \left[ \ln \left( \frac{m^2(\hat{\phi})}{\mu^2} \right) - N_\epsilon - \frac{3}{2} \right]. \quad (71)$$

As expected from previous works [17–26], the homogeneous contribution is constant even though the geometry is expanding. The  $N_\epsilon$  term is proportional to  $m^4(\hat{\phi})$ , so that we have three kinds of divergences: constant, quadratic in  $\hat{\phi}$  and quartic, which can be reabsorbed in the renormalization of the tree-level potential parameters  $V_0$ ,  $M^2$  and  $\lambda$ . This means that at the leading adiabatic order we obtain exactly the same divergences as in flat space-time and we do not need additional counterterms to renormalize the effective potential.

Following the minimal subtraction scheme  $\overline{\text{MS}}$ , we remove the terms proportional to  $N_\epsilon$ . Thus, we are left with the complete renormalized homogeneous effective potential

$$V_{\text{eff}}^h(\hat{\phi}) = V_0 + \frac{1}{2} M^2 \hat{\phi}^2 + \frac{\lambda}{4} \hat{\phi}^4 + \frac{m^4(\hat{\phi})}{64\pi^2} \left[ \ln \left( \frac{m^2(\hat{\phi})}{\mu^2} \right) - \frac{3}{2} \right], \quad (72)$$

which agrees with the standard result in flat space-time. Here, the physical mass  $M$  and coupling constant  $\lambda$  are defined at a given physical scale  $\mu$ . Since the renormalized effective potential is independent of the renormalization scale  $\mu$ ,  $M^2$  and the coupling constant should depend

on  $\mu$  according to the renormalization group equations

$$\beta(\lambda) \equiv \frac{d\lambda}{d(\log \mu)} = \frac{18\lambda^2}{(4\pi)^2} \quad (73)$$

$$\gamma_M(\lambda) \equiv \frac{d(\log M^2)}{d(\log \mu)} = \frac{6\lambda}{(4\pi)^2}.$$

On the other hand the inhomogeneous contribution is given in Fourier space by

$$V_1^i(\eta, \mathbf{p}) = \frac{1}{2} \int_0^{m^2(\hat{\phi})} dm^2 \Delta_i(\eta, \mathbf{p}), \quad (74)$$

which after introducing (66) reads

$$V_1^i(\eta, \mathbf{p}) = \frac{m^4(\hat{\phi})}{16\pi^2} (H_{\Phi+\Psi}(\eta, \mathbf{p}) + H_{\Phi-\Psi}(\eta, \mathbf{p})) \quad (75)$$

where  $H_{\Phi+\Psi}$  and  $H_{\Phi-\Psi}$  are given in Fourier space by

$$\left( \frac{m^4(\hat{\phi})}{16\pi^2} \right) H_{\Phi\pm\Psi}(\eta, \mathbf{p}) = \frac{1}{2} \int_0^{m^2(\hat{\phi})} dm^2 [\Delta_i^\Phi(\eta, \mathbf{p}) \pm \Delta_i^\Psi(\eta, \mathbf{p})] \left[ \frac{\Phi(\mathbf{p}) \pm \Psi(\mathbf{p})}{2} \right]. \quad (76)$$

As mentioned before, the inhomogeneous contributions are finite and therefore they are not affected by renormalization ambiguities. Notice also that since  $H_{\Phi\pm\Psi}(\eta, \mathbf{p})$  are the product of the metric potentials with a function of  $\mathbf{p}$ , the corresponding objects in position space  $H_{\Phi\pm\Psi}(\eta, \mathbf{x})$  will depend on the metric potentials in a non-local way<sup>3</sup>.

Finally, we can put the two terms together and obtain the complete result for the renormalized one-loop effective potential

$$V_{\text{eff}}(\hat{\phi}) = V(\hat{\phi}) + \frac{m^4(\hat{\phi})}{64\pi^2} \left[ \ln \left( \frac{m^2(\hat{\phi})}{\mu^2} \right) - \frac{3}{2} \right] + \frac{m^4(\hat{\phi})}{16\pi^2} [H_{\Phi+\Psi}(\eta, \mathbf{x}) + H_{\Phi-\Psi}(\eta, \mathbf{x})]. \quad (77)$$

Since the powers of  $\hat{\phi}$  that appear in the inhomogeneous terms are the same as those in the classical potential  $V(\hat{\phi})$ , it is possible to absorb the inhomogeneous contributions in a space-time dependent redefinition of the classical potential parameters

$$V_0 \rightarrow V_0 + \frac{M^4}{16\pi^2} [H_{\Phi+\Psi}(\eta, \mathbf{x}) + H_{\Phi-\Psi}(\eta, \mathbf{x})] \quad (78)$$

$$M^2 \rightarrow M^2 \left( 1 + \frac{3\lambda}{4\pi^2} [H_{\Phi+\Psi}(\eta, \mathbf{x}) + H_{\Phi-\Psi}(\eta, \mathbf{x})] \right) \quad (79)$$

<sup>3</sup> A non-local dependence does not imply a non-causal behaviour, provided the effects at a given space-time point are determined by events in its past light-cone

$$\lambda \rightarrow \lambda \left( 1 + \frac{9\lambda}{4\pi^2} [H_{\Phi+\Psi}(\eta, \mathbf{x}) + H_{\Phi-\Psi}(\eta, \mathbf{x})] \right). \quad (80)$$

This suggests that the Higgs VEV will also acquire space-time fluctuations.

### Higgs VEV

According to the discussion in Section II, once the effective potential is obtained, the value of the field for which

$$V_{\text{eff}}'(\hat{\phi}) = 0 \quad (81)$$

determines the vacuum expectation value  $\hat{\phi}$ . The inhomogeneous contributions will now induce a space-time dependence on  $\hat{\phi}$  which can be written as

$$\hat{\phi}(\eta, \mathbf{x}) = \hat{\phi}_0 + \Delta\hat{\phi}(\eta, \mathbf{x}), \quad (82)$$

where  $\hat{\phi}_0$  is the minimum of the potential in the absence of metric perturbations, but including the one-loop corrections, i.e.

$$V_{\text{eff}}^{\text{h}}'(\hat{\phi}_0) = 0, \quad (83)$$

then

$$\Delta\hat{\phi} = -\frac{V_1^{\text{h}}'(\hat{\phi}_0)}{V_{\text{eff}}^{\text{h}}''(\hat{\phi}_0)}. \quad (84)$$

Thus, the relative Higgs VEV variation is given by

$$\Delta_{\text{Higgs}} \equiv \frac{\Delta\hat{\phi}}{\hat{\phi}_0} = -\frac{3\lambda}{4\pi^2} (H_{\Phi+\Psi} + H_{\Phi-\Psi}). \quad (85)$$

The perturbation is therefore proportional to the Higgs self-coupling as expected at one-loop.

## VI. PARTICULAR CASES

### Non-expanding space-times: Solar System

Let us consider weak gravitational fields generated by static sources which could be a good approximation for the field produced by the Sun within the Solar System. For the corresponding space-time metric, we can take (1) with  $a(\eta) = 1$  and static potentials  $\Phi(\mathbf{x})$  and  $\Psi(\mathbf{x})$  which allow us to use the previous results. This simplifies the calculations in several of the steps discussed above. For instance, all the time integrals can be done in a straightforward way, there is no need to apply the Feynman trick since the  $\omega$ 's are all the same, and the coefficients  $R_{l,\log}^{\{\Phi,\Psi\}}$  are zero (see Appendix C).

The results for a non-expanding geometry read

$$R_l^\Phi(\eta) = R_{l,\text{pol}}^\Phi(\eta) = \frac{(-1)^{l+1}}{2(2l+1)!} (2l-1) \eta^{2l} \quad (86)$$

$$R_l^\Psi(\eta) = R_{l,\text{pol}}^\Psi(\eta) = \frac{(-1)^{l+1}}{2(2l+1)!} (2l+1) \eta^{2l}. \quad (87)$$

Closed analytical expression can be obtained for

$$\Delta_i^\Phi(\eta, \mathbf{p}) = \frac{m^2}{4\pi^2} \left\{ \frac{1}{2} \left[ \frac{2 \sin(p\eta)}{p\eta} - \cos(p\eta) - 1 \right] \right\} \quad (88)$$

$$\Delta_i^\Psi(\eta, \mathbf{p}) = \frac{m^2}{4\pi^2} \left\{ \frac{1}{2} [1 - \cos(p\eta)] \right\}. \quad (89)$$

Finally,  $H_{\Phi+\Psi}$  and  $H_{\Phi-\Psi}$  are given by

$$H_{\Phi+\Psi}(\eta, \mathbf{p}) = \left( \frac{\sin(p\eta)}{p\eta} - \cos(p\eta) \right) \left( \frac{\Phi(\mathbf{p}) + \Psi(\mathbf{p})}{2} \right) \quad (90)$$

$$H_{\Phi-\Psi}(\eta, \mathbf{p}) = \left( \frac{\sin(p\eta)}{p\eta} - 1 \right) \left( \frac{\Phi(\mathbf{p}) - \Psi(\mathbf{p})}{2} \right). \quad (91)$$

The above simple expressions allow us to get some insight on the way in which gravity is affecting the Higgs VEV. Let us assume for simplicity that  $\Phi = \Psi$  as is the case in pure General Relativity for a point-like source. Then  $H_{\Phi-\Psi} = 0$  and we only have the contribution from  $H_{\Phi+\Psi}$ . Let us now consider the limit of small  $p\eta$ , i.e. time intervals that are small compared to the wavelength of the corresponding Fourier mode of metric perturbations. Thus, expanding to leading order, we obtain

$$H_{\Phi+\Psi}(\eta, \mathbf{p}) = \frac{1}{3} p^2 \eta^2 \Psi(\mathbf{p}) + \dots = -\frac{{}^3R(\mathbf{p})\eta^2}{12} + \dots \quad (92)$$

where  ${}^3R(\mathbf{p}) = -4p^2\Psi(\mathbf{p})$  is the curvature of the spatial sections of constant time.

Thus, as we would expect from the strong equivalence principle, locally, i.e. for space-time points separated by small distances compared to the curvature radius of the manifold  $\eta \ll {}^3R(\mathbf{p})^{-1/2}$ , the new inhomogeneous contributions are negligibly small and the standard flat space-time result is recovered. In other words, locally the Higgs VEV can be considered as a constant, whereas for large separations  $\eta \geq {}^3R(\mathbf{p})^{-1/2}$  the gravitational effects cannot be neglected.

Although we have considered a particular coordinate choice in (1), corresponding to the longitudinal gauge, since in the absence of metric perturbations  $V_{\text{eff}}^{\text{h}}(\hat{\phi})$  is a constant, the Stewart-Walker lemma [35] guarantees that the obtained effective potential is gauge invariant. Observational implications of the results in (90) and (91) will be discussed in [36].

### Expanding space-times: Cosmology

Now we consider the case of a perturbed expanding universe with scale factor  $a(\eta)$  and constant metric perturbations  $\Phi(\mathbf{x})$  and  $\Psi(\mathbf{x})$ . In particular, we will concentrate in the matter dominated era in which the metric perturbations are constant both for sub-Hubble and super-Hubble modes. In addition, we will also provide results for super-Hubble modes in the radiation era and for general cosmologies with power-law scale factors, for which the metric perturbations are also constant.



TABLE I. Results for the  $\Psi$  contribution for several cosmologies whose scale factors as a function of the conformal time  $a(\eta)$  are given in the first column. The  $C_l$  coefficients defined in (93) and the  $A_{\text{cos}}$  and  $B_{\text{sin}}$  appearing in (95) are given for each case.  $\Omega_M$  and  $\Omega_R$  stand for the corresponding density parameters of matter and radiation.

Universe	$a(\eta)$	$C_l$	$A_{\text{cos}}$	$A_{\text{sin}}$
Matter	$\eta^2$	$2l/5$	$1/5$	$4/5$
Radiation	$\eta$	$2l/3$	$1/3$	$2/3$
Power law	$\eta^\delta$ ( $\delta \in \mathbb{N}^+$ )	$2l/(1+2\delta)$	$1/(1+2\delta)$	$2\delta/(1+2\delta)$

The results for the  $\Psi$  terms can be expressed as

$$R_l^\Psi(\eta) = R_{l,\text{pol}}^\Psi(\eta) = \frac{(-1)^{l+1}}{2(2l+1)!} (1 + C_l) \eta^{2l} a^2(\eta) \quad (93)$$

where  $C_l$  is given for each case in Table I. The  $R_{l,\log}^\Psi(\eta)$  terms vanish in all the cases studied (see Appendix C). The series in (93) can be summed to give

$$\Delta_i^\Psi(\eta, \mathbf{p}) = \frac{m^2}{4\pi^2} D(p\eta) \quad (94)$$

with

$$D(p\eta) = \frac{1}{2} \left[ (A_{\text{cos}} + A_{\text{sin}}) - A_{\text{cos}} \cos(p\eta) - A_{\text{sin}} \frac{\sin(p\eta)}{p\eta} \right] \quad (95)$$

where  $A_{\text{cos}}$  and  $A_{\text{sin}}$  are also given in Table I.

The  $\Phi$  contribution is more complicated since unlike the  $\Psi$  case, the  $R_{l,\log}^\Phi$  terms do not vanish. Then, the integration over the Feynman parameters  $\{x_i\}$  and the time integrals have to be performed by Taylor expanding the logarithm (see Appendix C). An exact analytical expression can be obtained for each order of the logarithm expansion given in terms of finite sums, which have to be computed numerically for practical purposes. On the other hand, the  $R_{l,\text{pol}}^\Phi$  terms cannot be written in terms of simple fractions like the  $R_{l,\text{pol}}^\Psi$  ones and they cannot be expressed in a closed form as in (93).

Nevertheless, for super-Hubble modes, with  $p\eta \ll 1$ , it is possible to get simple expressions to the leading order. Thus, assuming again  $\Phi = \Psi$  we get in a matter dominated universe

$$H_{\Phi+\Psi}(\eta, \mathbf{p}) = \frac{2}{9} p^2 \eta^2 \Psi(\mathbf{p}) + \dots \quad (96)$$

whereas for radiation domination

$$H_{\Phi+\Psi}(\eta, \mathbf{p}) = \frac{1}{4} p^2 \eta^2 \Psi(\mathbf{p}) + \dots \quad (97)$$

An extended discussion of these results and their cosmological implications will be given in [37].

## VII. DISCUSSION AND CONCLUSIONS

In this work, we have computed the one-loop corrections to the effective potential due to the self-interactions

of the Higgs field in a perturbed FRW background. For this aim, a complete orthonormal set of perturbative modes of the Klein-Gordon equation was obtained and an order-by-order dimensional regularization procedure was used. The results imply a space-time dependent correction to the effective potential. The fact that the inhomogeneous contributions come from the finite parts of the one-loop terms has important physical implications. First, it is well-known that unlike the local divergent terms, the finite ones will depend on the particular choice of vacuum state [15], which is equivalent to an election of the boundary conditions for the modes (Section IV). This is similar to the dependence on the vacuum state of the primordial spectrum of curvature perturbations generated during inflation. In our case, a particular vacuum choice would allow to cancel the new contributions in a particular point of space-time, for example, by building the vacuum with the set of mode solutions corresponding to a locally Minkowskian coordinate system at that particular point. However, such a cancellation will only be local and will not hold in regions beyond the curvature radius. In any case, as discussed above, once the vacuum state is chosen the effective potential is a gauge invariant quantity.

Another important aspect is that the finite contributions are sensitive to the large-scale properties of space-time, unlike the divergent parts which only depend on local properties of the manifold. In flat space-time, these finite parts are usually ignored since, being constant, they can be absorbed in the renormalized parameters of the theory. However, as shown in this work, in curved backgrounds they are in general space-time dependent. In principle, they can still be absorbed in a redefinition of the renormalized parameters, which will now turn into space-time dependent functions. In this sense, the results of the work presented here suggest a space-time dependence of the Higgs VEV and, as a consequence, of the particle masses. Nevertheless, since quantum field theory does not predict the value of any physical (renormalized) parameter, the actual variation could only be obtained from observations. The possible observable effects within the Solar System and on cosmological scales will be discussed in more detail in [36] and [37], respectively.

A complete calculation of the effective potential requires the introduction of the contributions from the

rest of fields that couple to the Higgs. In particular, the most important corrections are expected to come from the heaviest fields, i.e. the top quark and the W and Z bosons. However the quantization of such higher-spin fields in inhomogeneous space-times is technically more involved. Work is in progress in this direction.

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### Appendix A: Dimensional Regularization Formulae

The fundamental formula used in dimensional regularization in Euclidean space is [38, 39]

$$\int \frac{d^D \mathbf{k}}{(2\pi)^D} \frac{k^{2\alpha}}{(k^2 + m^2)^\beta} = m^{2(\alpha-\beta)} \left( \frac{m^2}{4\pi} \right)^{D/2} \frac{\Gamma(D/2 + \alpha) \Gamma(\beta - \alpha - D/2)}{\Gamma(\beta) \Gamma(D/2)}. \quad (\text{A1})$$

This expression has been used to compute  $\Delta_h$  in (51) in  $D = 3 - \epsilon$ . The left-hand side of the equation can be written as

$$\int \frac{d^D \mathbf{k}}{(2\pi)^D} \frac{k^{2\alpha}}{(k^2 + m^2)^\beta} = \frac{1}{(2\pi)^D} \frac{2\pi^{D/2}}{\Gamma(D/2)} \int_0^\infty dk \frac{k^{D-1} k^{2\alpha}}{(k^2 + m^2)^\beta} \quad (\text{A2})$$

then

$$\int_0^\infty dk \frac{k^{D-1} k^{2\alpha}}{(k^2 + m^2)^\beta} = \left[ \frac{1}{(2\pi)^D} \frac{2\pi^{D/2}}{\Gamma(D/2)} \right]^{-1} m^{2(\alpha-\beta)} \left( \frac{m^2}{4\pi} \right)^{D/2} \frac{\Gamma(D/2 + \alpha) \Gamma(\beta - \alpha - D/2)}{\Gamma(\beta) \Gamma(D/2)}. \quad (\text{A3})$$

On the other hand, for the  $\Delta_i$  term in (53), we have to deal with integrals of the following form

$$\int \frac{d^D \mathbf{k}}{(2\pi)^D} \frac{f(\mathbf{k} \cdot \mathbf{p})}{(k^2 + m^2)^\beta} \quad (\text{A4})$$

where  $f(\mathbf{k} \cdot \mathbf{p})$  is an analytical function. Taking the  $k_z$  direction along  $\mathbf{p}$ , we have  $f(\mathbf{k} \cdot \mathbf{p}) = f(k p \hat{x})$  with  $k = |\mathbf{k}|$ ,  $p = |\mathbf{p}|$  and  $\hat{x} = \cos(\theta)$ ,  $\theta$  being the angle between  $\mathbf{k}$  and  $\mathbf{p}$ . When using spherical coordinates  $\{\phi, \theta, \theta_2, \dots, \theta_{D-2}\}$ , the  $D$ -dimensional volume element can be expressed as

$$d^D \mathbf{k} = k^{D-1} \sin^{D-2}(\theta_{D-2}) \sin^{D-3}(\theta_{D-3}) \dots \sin(\theta) dk d\phi d\theta \dots d\theta_{D-2}. \quad (\text{A5})$$

The integrand of (A4) depends on  $\cos(\theta)$ , so we can integrate in all the angular variables but  $\theta$ . With that purpose, notice that the area of the unit  $(D-1)$ -sphere is

$$\overbrace{\int_0^\pi \dots \int_0^\pi}^{D-2} \int_0^{2\pi} \sin^{D-2}(\theta_{D-2}) \sin^{D-3}(\theta_{D-3}) \dots \sin^2(\theta_2) \sin(\theta) d\phi d\theta d\theta_2 \dots d\theta_{D-2} = \frac{2\pi^{D/2}}{\Gamma(D/2)}. \quad (\text{A6})$$

Since all the integrals involved can be factorized, the integration over all the angular variables but  $\theta$  is simply given by 1/2 of the previous result, i.e.  $\frac{\pi^{D/2}}{\Gamma(D/2)}$ . Therefore, equation (A4) can be expressed as

$$\int \frac{d^D \mathbf{k}}{(2\pi)^D} \frac{f(\mathbf{k} \cdot \mathbf{p})}{(k^2 + m^2)^\beta} = \frac{\pi^{D/2}}{\Gamma(D/2)} \int_0^\infty dk \frac{k^{D-1}}{(k^2 + m^2)^\beta} \hat{f}(kp). \quad (\text{A7})$$

where  $\hat{f}(kp) = \int_{-1}^1 d\hat{x} f(kp\hat{x})$ . Finally, Taylor expanding  $\hat{f}(kp)$ , the expression can be regularized order by order using equation (A3).

To regularize physical quantities like  $\Delta_h$  and  $\Delta_i$  two important aspects should be taken into consideration. First of all, the full physical expression should be computed in  $D$  dimensions, so that when taking  $D = 3 - \epsilon$  all the terms are expanded in  $\epsilon$ . Moreover, a physical scale  $\mu^\epsilon$  should be introduced to compensate the physical dimensions.

## Appendix B: $P_{k,l}^{\{\Phi, \Psi\}}$

In this appendix the exact expressions for the  $P_{k,l}^{\{\Phi, \Psi\}}(\eta)$  coefficients of equation (58) are given. First, let us separate these coefficients as

$$P_{k,l}^{\Phi}(\eta) = P_{k,l}^{\Phi,(0)}(\eta) + P_{k,l}^{\Phi,(1)}(\eta) + P_{k,l}^{\Phi,(2)}(\eta) \quad (\text{B1})$$

where the indices (0), (1), (2) stand for the contribution coming from  $P_k^{(0)}$  in (43),  $P_k^{(1)}$  in (44) and  $P_k^{(2)}$  in (45) respectively. The same definition applies for the terms  $P_{k,l}^{\Psi}$ .

The  $l = 0$  coefficients are given by

$$P_{k,0}^{\Phi}(\eta) = 0 \quad (\text{B2})$$

$$P_{k,0}^{\Psi}(\eta) = P_{k,0}^{\Psi,(0)}(\eta) = \frac{k^2}{\omega^2(\eta)} \left( D - \frac{k^2}{\omega^2(\eta)} \right) \quad (\text{B3})$$

For  $l > 0$ , we have

$$P_{k,l}^{\Phi}(\eta) = P_{k,l}^{\Phi,(1)}(\eta) + P_{k,l}^{\Phi,(2)}(\eta) \quad (\text{B4})$$

$$P_{k,l}^{\Psi}(\eta) = P_{k,l}^{\Psi,(1)}(\eta) + P_{k,l}^{\Psi,(2)}(\eta) \quad (\text{B5})$$

with

$$P_{k,l}^{\Phi,(1)}(\eta) = (-1)^{l+1} \frac{2l}{(2l+1)!} k^{2(l+1)} \int_0^\eta d\eta' \left( \prod_{i=1}^{2l-1} \int_{\eta'}^\eta \frac{d\eta_i}{\omega(\eta_i)} \right) \left[ \frac{\omega(\eta')}{\omega^3(\eta)} - \frac{1}{\omega(\eta)\omega(\eta')} \right] \quad (\text{B6})$$

$$P_{k,l}^{\Psi,(1)}(\eta) = (-1)^{l+1} \frac{2l}{(2l+1)!} k^{2(l+1)} \int_0^\eta d\eta' \left( \prod_{i=1}^{2l-1} \int_{\eta'}^\eta \frac{d\eta_i}{\omega(\eta_i)} \right) \left[ \frac{k^2}{\omega^3(\eta)\omega(\eta')} + \frac{k^2}{\omega(\eta)\omega^3(\eta')} - \frac{2}{\omega(\eta)\omega(\eta')} \right] \quad (\text{B7})$$

$$P_{k,l}^{\Phi,(2)}(\eta) = (-1)^{l+1} \frac{2l}{(2l+1)!} k^{2(l+1)} \int_0^\eta d\eta' \int_0^{\eta'} d\eta'' \left( \prod_{i=1}^{2l-2} \int_{\eta''}^\eta \frac{d\eta_i}{\omega(\eta_i)} \right) \left[ \frac{(2l-1)\omega(\eta'')}{\omega(\eta)\omega^3(\eta')} - \frac{(2l+1)\omega(\eta'')}{k^2\omega(\eta)\omega(\eta')} \right] \quad (\text{B8})$$

$$P_{k,l}^{\Psi,(2)}(\eta) = (-1)^{l+1} \frac{2l}{(2l+1)!} k^{2(l+1)} \int_0^\eta d\eta' \int_0^{\eta'} d\eta'' \left( \prod_{i=1}^{2l-2} \int_{\eta''}^\eta \frac{d\eta_i}{\omega(\eta_i)} \right) \left[ \frac{(2l-1)k^2}{\omega(\eta)\omega^3(\eta')\omega(\eta'')} - \frac{(2l+1)}{\omega(\eta)\omega(\eta')\omega(\eta'')} \right] \quad (\text{B9})$$

The integral over  $k$  of all these terms can be regularized with the expressions given in Appendix A after applying the generalized Feynmann trick discussed in Section V. After regularization, we are left with two terms: one polynomial in  $\eta$ , the other one logarithmic in  $\eta$ . The integration over the Feynman parameters  $\{x_i\}$  and the time integrals can be done following the procedure discussed in Appendix C.

## Appendix C: Integration over $\{x_i\}$ and $\{\eta_i\}$

This appendix shows how to compute the integrals over  $\{x_i\}$  and  $\{\eta_i\}$  appearing in the  $R_l^{\{\Phi, \Psi\}}$  coefficients in (67). These terms have the general form

$$\overbrace{\int d\eta_1 \cdots \int d\eta_{2N}}^{2N} \overbrace{\int_0^1 \frac{dx_1}{\sqrt{x_1}} \cdots \int_0^1 \frac{dx_{2N+1}}{\sqrt{x_{2N+1}}}}^{2N+1} \delta \left( \sum_{k=1}^{2N+1} x_k - 1 \right) \left\{ \text{Pol}_1(\{x_i\}, \{\eta_i\}) + \log \left[ \sum_{k=1}^{2N+1} x_k a^2(\eta_k) \right] \text{Pol}_2(\{x_i\}, \{\eta_i\}) \right\} \quad (\text{C1})$$

where the logarithmic contribution is included in the  $R_{l,\log}^{\{\Phi, \Psi\}}$  part of (68) whereas the pure polynomial one coming from  $\text{Pol}_1$  is included in  $R_{l,\text{pol}}^{\{\Phi, \Psi\}}$ . Notice that we have redefined  $2l$  appearing in expression (67), namely the power of  $p$ , to be  $2N$  in (C1) in order to highlight its importance in the following discussion. Since the polynomials only introduce trivial modifications of the following formulae, let us focus on the expression

$$\overbrace{\int d\eta_1 \cdots \int d\eta_{2N}}^{2N} \overbrace{\int_0^1 \frac{dx_1}{\sqrt{x_1}} \cdots \int_0^1 \frac{dx_{2N+1}}{\sqrt{x_{2N+1}}}}^{2N+1} \delta \left( \sum_{k=1}^{2N+1} x_k - 1 \right) \log \left[ \sum_{k=1}^{2N+1} x_k a^2(\eta_k) \right]. \quad (\text{C2})$$

There are  $2N + 1$  variables  $x_i$  from the Feynmann trick and all of them are integrated from 0 to 1. There are also  $2N + 1$  time variables  $\eta_i$ , but only  $2N$  of them are integrated. In particular,  $\eta_{2N+1}$  is not integrated. In order to recover the expressions given in the text, we have renamed  $\eta$  as  $\eta_{2N+1}$ ,  $\eta'$  as  $\eta_{2N}$  and  $\eta''$  as  $\eta_{2N-1}$ . From the general expression (C2), it is straightforward to prove that for  $a(\eta) = 1$ , the logarithm vanishes since  $\sum_{k=1}^{2N+1} x_k = 1$ . Therefore,  $R_{l, \log}^{\{\Phi, \Psi\}} = 0$  in non-expanding space-times.

First, we deal with the integration over the  $\{x_i\}$ . Defining new variables  $y_i^2 = x_i$  for  $i = 1, \dots, 2N + 1$ , this integration can be written over the  $2N$ -sphere

$$\int_0^1 \frac{dx_1}{\sqrt{x_1}} \cdots \int_0^1 \frac{dx_{2N+1}}{\sqrt{x_{2N+1}}} \delta\left(\sum_{k=1}^{2N+1} x_k - 1\right) = 2^{2N} \int_{S^{2N}} d^{2N}\Omega. \quad (C3)$$

Then, the logarithm can be expressed as

$$\log \left[ \sum_{k=1}^{2N+1} y_k^2 a^2(\eta_k) \right] = \log [a^2(\eta_{2N+1})] + \log \left[ 1 + \sum_{k=1}^{2N} y_k^2 \left( \frac{a^2(\eta_k)}{a^2(\eta_{2N+1})} - 1 \right) \right] \quad (C4)$$

where we have used that  $y_{2N+1}^2 = 1 - \sum_{k=1}^{2N} y_k^2$ . The first logarithm on the right-hand side is the usual logarithm of the scale factor which appears in dimensional regularization in a FRW metric and it cancels out at the end. On the other hand, since  $\eta_{2N+1}$  is an upper limit in all the time integrations (see next subsection), we have  $\eta_k \leq \eta_{2N+1}$  for  $k = 1, \dots, 2N$ . Thus, considering expanding universes, the argument of the logarithm is of the form  $1 + x$  with  $-1 < x \leq 1$ . Hence, it can be Taylor expanded as

$$\log \left[ 1 + \sum_{k=1}^{2N} y_k^2 \left( \frac{a^2(\eta_k)}{a^2(\eta_{2N+1})} - 1 \right) \right] = \sum_{j=1}^{\infty} \frac{(-1)^{j+1}}{j} \left[ \sum_{k=1}^{2N} y_k^2 \left( \frac{a^2(\eta_k)}{a^2(\eta_{2N+1})} - 1 \right) \right]^j \quad (C5)$$

where the last factor on the right-hand side can also be expanded using the multinomial theorem

$$\left[ \sum_{k=1}^{2N} y_k^2 \left( \frac{a^2(\eta_k)}{a^2(\eta_{2N+1})} - 1 \right) \right]^j = \sum_{\substack{l_1, l_2, \dots, l_{2N}=0 \\ \sum_{i=1}^{2N} l_i = j}}^j \frac{j!}{l_1! l_2! \cdots l_{2N}!} \prod_{m=1}^{2N} \left[ y_m^2 \left( \frac{a^2(\eta_m)}{a^2(\eta_{2N+1})} - 1 \right) \right]^{l_m} \quad (C6)$$

Therefore, the integration over the  $2N$ -sphere reduces to an integration of this kind

$$\int_{S^{2N}} d^{2N}\Omega y_1^{2l_1} y_2^{2l_2} \cdots y_{2N}^{2l_{2N}} = \frac{\sqrt{\pi} \prod_{i=1}^{2N} \Gamma(\frac{1}{2} + l_i)}{2^{2N} \Gamma(N + \frac{1}{2} + \sum_{i=1}^{2N} l_i)} \equiv \frac{1}{2^{2N}} \Gamma[\{l_i\}, 2N]. \quad (C7)$$

Then

$$2^{2N} \int_{S^{2N}} d^{2N}\Omega \log \left[ 1 + \sum_{k=1}^{2N} y_k^2 \left( \frac{a^2(\eta_k)}{a^2(\eta_{2N+1})} - 1 \right) \right] = \sum_{j=1}^{\infty} \frac{(-1)^{j+1}}{j} \sum_{\substack{l_1, l_2, \dots, l_{2N}=0 \\ \sum_{i=1}^{2N} l_i = j}}^j \frac{j!}{l_1! l_2! \cdots l_{2N}!} \Gamma[\{l_i\}, 2N] \prod_{m=1}^{2N} \left( \frac{a^2(\eta_m)}{a^2(\eta_{2N+1})} - 1 \right)^{l_m} \quad (C8)$$

Applying the binomial theorem to the last factors,

$$\left( \frac{a^2(\eta_m)}{a^2(\eta_{2N+1})} - 1 \right)^{l_m} = \sum_{i_m=0}^{l_m} (-1)^{l_m-i_m} \binom{l_m}{i_m} \left[ \frac{a^2(\eta_m)}{a^2(\eta_{2N+1})} \right]^{i_m}, \quad (C9)$$

and gathering all the results we get

$$2^{2N} \int_{S^{2N}} d^{2N}\Omega \log \left[ 1 + \sum_{k=1}^{2N} y_k^2 \left( \frac{a^2(\eta_k)}{a^2(\eta_{2N+1})} - 1 \right) \right] = - \sum_{j=1}^{\infty} \sum_{\substack{l_1, l_2, \dots, l_{2N}=0 \\ \sum_{i=1}^{2N} l_i = j}}^j \frac{(j-1)!}{l_1! l_2! \cdots l_{2N}!} \Gamma[\{l_i\}, 2N] \sum_{i_1, i_2, \dots, i_{2N}=0}^{l_1, l_2, \dots, l_{2N}} (-1)^{\sum_{m=1}^{2N} i_m} \prod_{m=1}^{2N} \binom{l_m}{i_m} \left[ \frac{a^2(\eta_m)}{a^2(\eta_{2N+1})} \right]^{i_m}. \quad (C10)$$

Finally, the time integrations can be done in a straightforward way since the dependence on  $\eta_m$  of the scale factor is polynomial for the cosmologies considered in this work.

$$R_{l,\log}^\Psi = 0 \text{ for all cosmologies}$$

In Section VI, it is mentioned that the  $R_{l,\log}^\Psi$  coefficients are all zero for all the cases considered. In fact, these expressions vanish not because of the integration over  $\{x_i\}$  but because the polynomial  $\text{Pol}_2(\{x_i\}, \{\eta_i\})$  in (C1) is zero for the  $\Psi$  contribution. This can be shown by summing the already-regularized expression for (B5). Although the limits of integration are apparently different in each of the terms (B7), (B9); the region of integration is the same. For instance, the first integral can be written as

$$\int_0^\eta d\eta' \left( \prod_{i=1}^{2l-1} \int_{\eta'}^\eta d\eta_i \right) = \int_0^\eta d\eta' \left( \prod_{i=1}^{2l-1} \int_0^\eta d\eta_i \theta(\eta_i - \eta') \right), \quad (\text{C11})$$

where  $\theta$  is the step function, while

$$\int_0^\eta d\eta' \int_0^{\eta'} d\eta'' \left( \prod_{i=1}^{2l-2} \int_{\eta''}^\eta d\eta_i \right) = \int_0^\eta d\eta' \int_0^\eta d\eta'' \theta(\eta' - \eta'') \left( \prod_{i=1}^{2l-2} \int_0^\eta d\eta_i \theta(\eta_i - \eta'') \right). \quad (\text{C12})$$

Then, redefining in the last integral  $\eta'$  as  $\eta_{2l-1}$  and  $\eta''$  as  $\eta'$ , both integrals have the same form

$$\overbrace{\int_0^\eta d\eta_1 \cdots \int_0^\eta d\eta_{2N}}^{2N} \prod_{i=1}^{2N-1} \theta(\eta_i - \eta_{2N}). \quad (\text{C13})$$

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